Complete Relativity

Miko-pedia AI

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Contents

1 Special Relativity and Flat Spacetime

1.1 Core ideas

Special relativity replaces absolute time with Minkowski spacetime. Events are related by Lorentz transformations, proper time is invariant, and energy-momentum forms a four-vector. Causality is controlled by light cones, not by simultaneity in a chosen frame.

For review, be able to use Lorentz transformations, compute proper time, manipulate four-vectors, and classify intervals as timelike, null, or spacelike. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

1.2 Mathematical spine

$$ds^2 = -c^2dt^2 + d\boldsymbol{x}^2, \qquad p^\mu = (E/c, \boldsymbol{p}), \qquad E^2 = p^2c^2 + m^2c^4$$

Section summary Flat spacetime unifies space, time, energy, and momentum.

2 Manifolds

2.1 Core ideas

General relativity needs coordinate-independent geometry. A manifold is a space that locally looks like $\mathbb{R}^n$ but can be curved globally. Vectors live in tangent spaces, covectors in dual spaces, and tensors encode geometric objects independent of coordinates.

For review, be able to distinguish coordinates from geometry, transform tensor components, use tangent vectors and one-forms, and interpret metrics as inner products. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

2.2 Mathematical spine

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu, \qquad V^{\mu'} = \frac{\partial x^{\mu'}}{\partial x^\nu}V^\nu$$

Section summary Manifolds provide the coordinate-free stage for spacetime.

3 Curvature

3.1 Core ideas

Curvature measures the failure of vectors to return unchanged after parallel transport. The connection defines covariant derivatives and geodesics; the Riemann tensor measures curvature; Ricci curvature and the scalar curvature are contractions used in Einstein's equation.

For review, be able to compute Christoffel symbols for simple metrics, write geodesic equations, and interpret Riemann/Ricci curvature geometrically. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

3.2 Mathematical spine

$$\Gamma^\rho_{\mu\nu} = \frac{1}{2}g^{\rho\sigma}(\partial_\mu g_{\nu\sigma} + \partial_\nu g_{\mu\sigma} - \partial_\sigma g_{\mu\nu}), \qquad R^\rho{}_{\sigma\mu\nu}V^\sigma = [\nabla_\mu, \nabla_\nu]V^\rho$$

Section summary Curvature is encoded in how covariant derivatives fail to commute.

4 Gravitation

4.1 Core ideas

Einstein gravity identifies gravity with spacetime curvature sourced by stress-energy. Freely falling bodies follow geodesics; the Newtonian potential appears as a weak-field, slow-motion limit. Conservation of stress-energy follows from geometry through the Bianchi identity.

For review, be able to state Einstein's equation, recover Newtonian gravity in weak fields, identify stress-energy components, and use geodesics for motion. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

4.2 Mathematical spine

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \qquad \frac{d^2 x^\mu}{d\tau^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = 0$$

Section summary Matter tells spacetime how to curve; curved spacetime tells matter how to move.

5 The Schwarzschild Solution

5.1 Core ideas

The Schwarzschild metric describes the exterior field of a static spherical mass. It predicts gravitational redshift, light bending, perihelion precession, black hole horizons, and the structure of radial and circular geodesics.

For review, be able to use the Schwarzschild radius, identify horizon vs singularity, compute qualitative orbits, and explain classic weak-field tests. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

5.2 Mathematical spine

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right) c^2 dt^2 + \left(1 - \frac{2GM}{rc^2}\right)^{-1} dr^2 + r^2 d\Omega^2$$

Section summary Schwarzschild spacetime is the basic laboratory for relativistic gravity.

6 More General Black Holes / Perturbations and Gravitational Waves

6.1 Core ideas

Rotating and charged black holes add angular momentum and charge; astrophysical black holes are described mainly by Kerr geometry. Small metric perturbations propagate as gravitational waves with two tensor polarizations. Detectors measure strain from compact binary motion.

For review, be able to recognize Kerr parameters, describe horizons and ergospheres qualitatively, derive the linear wave equation, and connect quadrupole radiation to binary inspiral. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

6.2 Mathematical spine

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \qquad \Box \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4} T_{\mu\nu}, \qquad h \sim \Delta L / L$$

Section summary Black holes and gravitational waves are dynamical strong-field predictions of GR.

7 Cosmology

7.1 Core ideas

Relativistic cosmology assumes large-scale homogeneity and isotropy, leading to the FLRW metric and Friedmann equations. Matter, radiation, curvature, and dark energy determine expansion. Observations of redshift, CMB, supernovae, and structure constrain the model.

For review, be able to write the FLRW metric, use scale factor and redshift, interpret density parameters, and connect Friedmann equations to cosmic history. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

7.2 Mathematical spine

$$ds^2 = -c^2dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right], \qquad H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}$$

Section summary Cosmology applies GR to the expanding universe.