Complete Quantum Field Theory

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Contents

1 Part I: Feynman Diagrams and Quantum Electrodynamics

1.1 Core ideas

QFT combines quantum mechanics, special relativity, and many-particle physics. Particles are excitations of fields, interactions are local terms in a Lagrangian, and perturbation theory organizes scattering amplitudes as Feynman diagrams.

For review, be able to explain why fields replace fixed-particle wave functions, identify propagators and vertices, and connect amplitudes to observables. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

1.2 Mathematical spine

$$S = \int d^4x\, \mathcal{L}, \qquad \langle f|S|i\rangle = \delta_{fi} + i(2\pi)^4\delta^4(P_f - P_i)\mathcal{M}$$

Section summary QFT computes relativistic quantum processes through fields and amplitudes.

2 The Klein-Gordon Field

2.1 Core ideas

The Klein-Gordon field is the simplest relativistic quantum field. Quantization turns each momentum mode into a harmonic oscillator and introduces creation and annihilation operators. The propagator is the Green function for relativistic propagation.

For review, be able to derive the equation of motion, quantize modes, interpret particles and antiparticles, and write the scalar propagator. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

2.2 Mathematical spine

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\,\partial^\mu\phi - \frac{1}{2}m^2\phi^2, \qquad (\Box + m^2)\phi = 0, \qquad \Delta_F(p) = \frac{i}{p^2 - m^2 + i\epsilon}$$

Section summary Scalar fields show how particles emerge from quantized modes.

3 The Dirac Field

3.1 Core ideas

The Dirac field describes spin-1/2 fermions. Gamma matrices linearize the relativistic dispersion relation, spinors carry Lorentz representation data, and anticommutation enforces Pauli exclusion and positive energy.

For review, be able to use the Dirac equation, spinor completeness, conserved current, and the fermion propagator. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

3.2 Mathematical spine

$$\mathcal{L} = \bar{\psi}(i\gamma^\mu\partial_\mu - m)\psi, \qquad (i\gamma^\mu\partial_\mu - m)\psi = 0, \qquad S_F(p) = \frac{i(\not{p} + m)}{p^2 - m^2 + i\epsilon}$$

Section summary Dirac fields are relativistic quantum fields for fermions.

4 Interacting Fields and Feynman Diagrams

4.1 Core ideas

Interactions make fields scatter and decay. Perturbation theory expands time-ordered correlation functions in coupling constants; Wick's theorem reduces products of free fields to propagators and vertices.

For review, be able to derive simple Feynman rules, count symmetry factors, and relate diagrams to powers of coupling. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

4.2 Mathematical spine

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{int}}, \qquad Z[J] = \int \mathcal{D}\phi\, e^{iS[\phi] + i\int J\phi}$$

Section summary Feynman diagrams are bookkeeping for perturbative correlation functions.

5 Elementary Processes of QED

5.1 Core ideas

QED couples the Dirac field to the electromagnetic gauge field. Tree-level processes such as electron-muon scattering, annihilation, pair production, and Compton scattering illustrate spinor algebra, gauge invariance, and cross-section calculations.

For review, be able to write QED Feynman rules, use Ward identities conceptually, and compute the structure of basic amplitudes. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

5.2 Mathematical spine

$$\mathcal{L}_{\text{QED}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(i\gamma^\mu D_\mu - m)\psi, \qquad D_\mu = \partial_\mu + ieA_\mu$$

Section summary QED is the cleanest example of a quantum gauge theory.

6 Radiative Corrections

6.1 Core ideas

Loops correct masses, charges, magnetic moments, and scattering amplitudes. They introduce divergences that must be regularized and renormalized. Physical predictions depend on measured parameters at a scale, not on bare quantities.

For review, be able to identify self-energy, vertex, and vacuum polarization corrections, explain regularization, and interpret running charge. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

6.2 Mathematical spine

$$\alpha(q^2) = \frac{\alpha(\mu^2)}{1 - \Pi(q^2, \mu^2)}, \qquad g - 2 \text{ starts at } \frac{\alpha}{2\pi}$$

Section summary Radiative corrections make QFT predictive beyond tree level.

7 Part II: Renormalization

7.1 Core ideas

Renormalization separates short-distance unknowns from long-distance predictions. Counterterms absorb divergences, while renormalized couplings depend on scale. Relevant, marginal, and irrelevant operators explain why low-energy physics can be universal.

For review, be able to classify operators by dimension, state what counterterms do, and distinguish bare from renormalized parameters. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

7.2 Mathematical spine

$$\mathcal{L} = \mathcal{L}_{\text{ren}} + \mathcal{L}_{\text{counter}}, \qquad \mu\frac{dg}{d\mu} = \beta(g)$$

Section summary Renormalization is scale-dependent bookkeeping of physical parameters.

8 Functional Methods

8.1 Core ideas

Path integrals compute generating functionals for correlation functions. Sources generate expectation values, effective actions generate one-particle-irreducible vertices, and saddle points connect quantum field theory to classical field equations.

For review, be able to use generating functionals, functional derivatives, Wick rotation, and effective action concepts. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

8.2 Mathematical spine

$$Z[J] = \int \mathcal{D}\phi \, \exp\left(iS[\phi] + i\int J\phi \, d^4x\right), \qquad \langle \phi(x)\phi(y)\rangle = \frac{1}{i^2 Z}\frac{\delta^2 Z}{\delta J(x)\delta J(y)}$$

Section summary Functional methods make symmetries and correlation functions systematic.

9 Systematics of Renormalization

9.1 Core ideas

Renormalization requires all counterterms allowed by symmetries. Power counting tells which diagrams diverge; schemes such as dimensional regularization and minimal subtraction define finite parts. Symmetry identities constrain counterterms.

For review, be able to perform superficial degree-of-divergence counting, explain schemes, and use symmetry to restrict terms. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

9.2 Mathematical spine

$$D = 4L - 2I_B - I_F + \sum_v d_v, \qquad \frac{1}{\epsilon} \text{ poles are absorbed into counterterms}$$

Section summary The structure of divergences is organized by dimension and symmetry.

10 Renormalization Group

10.1 Core ideas

The renormalization group describes how theories change with scale. Fixed points, beta functions, anomalous dimensions, and running couplings explain asymptotic freedom, critical phenomena, and effective field theory.

For review, be able to interpret beta functions, fixed points, relevant perturbations, and running couplings. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

10.2 Mathematical spine

$$\mu \frac{dg_i}{d\mu} = \beta_i(\{g\}), \qquad \mathcal{O}(\lambda x) = \lambda^{-\Delta}\mathcal{O}(x)$$

Section summary RG flow is the map of physics across length scales.

11 Critical Phenomena

11.1 Core ideas

QFT methods apply to continuous phase transitions because long-wavelength fluctuations dominate near criticality. Universality classes depend on symmetry, dimension, and order-parameter components rather than microscopic details.

For review, be able to connect Landau-Ginzburg fields to critical exponents, use correlation length, and explain universality. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

11.2 Mathematical spine

$$\mathcal{L} = \frac{1}{2}(\partial\phi)^2 + \frac{1}{2}r\phi^2 + \frac{u}{4!}\phi^4, \qquad \xi \sim |T - T_c|^{-\nu}$$

Section summary Critical phenomena are statistical field theories near fixed points.

12 Part III: Non-Abelian Gauge Theories

12.1 Core ideas

Non-Abelian gauge theories generalize electromagnetism by making gauge fields carry the charge they mediate. This self-interaction is responsible for asymptotic freedom, confinement, and the structure of the Standard Model.

For review, be able to explain local gauge symmetry, covariant derivatives, field strengths, and gauge-boson self-interactions. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

12.2 Mathematical spine

$$D_\mu = \partial_\mu - igA^a_\mu T^a, \qquad F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$$

Section summary Non-Abelian gauge symmetry is the backbone of modern particle physics.

13 Yang-Mills Theory

13.1 Core ideas

Pure Yang-Mills theory contains only non-Abelian gauge fields. Gauge fixing and ghosts are needed for covariant quantization. The theory is classically simple but quantum mechanically rich, with confinement and topological sectors.

For review, be able to write the Yang-Mills action, explain gauge fixing, ghosts, and why gauge redundancy is not a physical degree of freedom. Keep the physical question visible:

identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

13.2 Mathematical spine

$$\mathcal{L}_{\rm YM} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu}, \qquad \mathcal{L}_{\rm gf} = -\frac{1}{2\xi}(\partial_\mu A^{a\mu})^2$$

Section summary Yang-Mills theory is the core non-Abelian gauge field theory.

14 QCD

14.1 Core ideas

QCD is Yang-Mills theory with quarks in the fundamental representation of color. It is weakly coupled at high energy and strongly coupled at low energy. Chiral symmetry, confinement, jets, and hadrons are all consequences of color dynamics.

For review, be able to state QCD Lagrangian, explain asymptotic freedom and confinement, and connect quarks/gluons to hadrons and jets. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

14.2 Mathematical spine

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4}G^a_{\mu\nu}G^{a\mu\nu} + \bar{q}(i\gamma^\mu D_\mu - m)q, \qquad \beta(g) < 0$$

Section summary QCD is the quantum field theory of strong interactions.

15 Electroweak Theory

15.1 Core ideas

Electroweak theory unifies weak and electromagnetic interactions through $SU(2)_L \times U(1)_Y$ gauge symmetry. The Higgs mechanism breaks it to electromagnetism, giving masses to W, Z, and fermions while leaving the photon massless.

For review, be able to write the symmetry breaking pattern, identify charged and neutral currents, and explain masses from the Higgs vacuum expectation value. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

15.2 Mathematical spine

$$SU(2)_L \times U(1)_Y \to U(1)_{\rm em}, \qquad M_W = \frac{1}{2}gv, \qquad M_Z = \frac{1}{2}\sqrt{g^2 + g'^2}\,v$$

Section summary Electroweak theory is chiral gauge theory with spontaneous symmetry breaking.

16 Anomalies and Beyond

16.1 Core ideas

Anomalies occur when a classical symmetry fails after quantization. Gauge anomalies must cancel for consistency; global anomalies can have physical consequences. Effective field theory extends the Standard Model by higher-dimension operators.

For review, be able to distinguish global and gauge anomalies, state anomaly cancellation, and use EFT as a language for new physics. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

16.2 Mathematical spine

$$\partial_\mu j_5^\mu = \frac{g^2}{16\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}, \qquad \mathcal{L}_{\mathrm{EFT}} = \mathcal{L}_{\mathrm{SM}} + \sum_i \frac{c_i}{\Lambda^{d_i-4}}\mathcal{O}_i$$

Section summary Anomalies and EFT show how quantum consistency and high scales shape low-energy physics.