Complete Electromagnetism

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Contents

1 Vector Analysis

1.1 Core ideas

Electromagnetism is written in the language of fields. Gradients describe how scalar potentials change, divergence measures sources and sinks, curl measures circulation, and integral theorems connect local differential laws to global flux and circulation laws. Delta functions represent point charges and currents cleanly.

For review, be able to move between differential and integral forms, use divergence and Stokes' theorems, interpret boundary normals, and work in Cartesian, cylindrical, and spherical symmetry. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

1.2 Mathematical spine

$$\int_V \nabla \cdot \boldsymbol{A}\, d^3x = \oint_{\partial V} \boldsymbol{A} \cdot d\boldsymbol{a}, \qquad \int_S (\nabla \times \boldsymbol{A}) \cdot d\boldsymbol{a} = \oint_{\partial S} \boldsymbol{A} \cdot d\boldsymbol{\ell}$$

Section summary Vector analysis is the grammar of field laws.

2 Electrostatics

2.1 Core ideas

Electrostatics studies fields produced by stationary charge. Coulomb's law gives the field of point charges, superposition builds general charge distributions, and Gauss's law solves high-symmetry problems. Electrostatic fields are conservative, so work is path independent and energy can be stored in the field.

For review, be able to compute fields from charge distributions, choose Gaussian surfaces, relate force, field, and potential energy, and use boundary conditions at charged surfaces. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

2.2 Mathematical spine

$$\boldsymbol{E}(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0} \int \rho(\boldsymbol{r}') \frac{\boldsymbol{r} - \boldsymbol{r}'}{|\boldsymbol{r} - \boldsymbol{r}'|^3}\, d^3r', \qquad \nabla \cdot \boldsymbol{E} = \rho/\epsilon_0$$

Section summary Static charge creates conservative electric fields.

3 Potentials

3.1 Core ideas

Because electrostatic fields have zero curl, they can be written as the gradient of a scalar potential. Poisson and Laplace equations turn field problems into boundary value problems. Green functions, image charges, and separation of variables are the main tools for conductors and prescribed boundaries.

For review, be able to solve simple Poisson/Laplace problems, use uniqueness, apply image charges, and interpret equipotentials and boundary conditions. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

3.2 Mathematical spine

$$\boldsymbol{E} = -\nabla V, \qquad \nabla^2 V = -\rho/\epsilon_0, \qquad V(\boldsymbol{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\rho(\boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|} \, d^3r'$$

Section summary Potentials make electrostatics a boundary value problem.

4 Electric Fields in Matter

4.1 Core ideas

Matter polarizes in electric fields. Bound charge is described by polarization $\boldsymbol{P}$, while the displacement field $\boldsymbol{D}$ separates free charge from bound response. Linear dielectrics are summarized by permittivity, but the physical picture is microscopic dipoles plus boundary conditions.

For review, be able to compute bound charge, use $\boldsymbol{D}$ for free charge, apply dielectric boundary conditions, and compare vacuum and material energy densities. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

4.2 Mathematical spine

$$\rho_b = -\nabla \cdot \boldsymbol{P}, \qquad \boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P}, \qquad \nabla \cdot \boldsymbol{D} = \rho_{\text{free}}$$

Section summary Dielectrics modify fields through polarization.

5 Magnetostatics

5.1 Core ideas

Magnetostatics describes steady currents and time-independent magnetic fields. The Biot–Savart law gives fields from currents, Ampere's law solves symmetric systems, and $abla \cdot \boldsymbol{B} = 0$ states that magnetic monopoles are absent in classical electromagnetism.

For review, be able to compute fields of wires, loops, and solenoids, choose Amperian loops, use vector potential, and understand magnetic force on currents. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

5.2 Mathematical spine

$$\boldsymbol{B}(\boldsymbol{r}) = \frac{\mu_0}{4\pi} \int \frac{\boldsymbol{J}(\boldsymbol{r}') \times (\boldsymbol{r} - \boldsymbol{r}')}{|\boldsymbol{r} - \boldsymbol{r}'|^3} \, d^3r', \qquad \nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J}$$

Section summary Steady currents produce divergence-free magnetic fields.

6 Magnetic Fields in Matter

6.1 Core ideas

Magnetized matter contains microscopic current loops. Magnetization $\boldsymbol{M}$ produces bound volume and surface currents, while $\boldsymbol{H}$ separates free current from material response. Paramagnets, diamagnets, and ferromagnets differ in how $\boldsymbol{M}$ responds to applied field.

For review, be able to compute bound currents, use $\boldsymbol{H}$ boundary conditions, distinguish susceptibility regimes, and recognize hysteresis as nonlinear material response. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

6.2 Mathematical spine

$$\boldsymbol{J}_b = \nabla \times \boldsymbol{M}, \qquad \boldsymbol{K}_b = \boldsymbol{M} \times \hat{\boldsymbol{n}}, \qquad \boldsymbol{H} = \boldsymbol{B}/\mu_0 - \boldsymbol{M}$$

Section summary Magnetic media are handled by separating free and bound currents.

7 Electrodynamics

7.1 Core ideas

Time-dependent electric and magnetic fields generate one another. Faraday induction and Maxwell's displacement current complete the static laws and enforce charge conservation. Maxwell's equations are the compact local statement of classical electromagnetism.

For review, be able to write all four Maxwell equations, convert between integral and differential forms, derive continuity, and identify quasistatic limits. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

7.2 Mathematical spine

$$\nabla \cdot \boldsymbol{E} = \rho/\epsilon_0, \quad \nabla \cdot \boldsymbol{B} = 0, \quad \nabla \times \boldsymbol{E} = -\partial_t \boldsymbol{B}, \quad \nabla \times \boldsymbol{B} = \mu_0 \boldsymbol{J} + \mu_0 \epsilon_0 \partial_t \boldsymbol{E}$$

Section summary Maxwell's equations unify static and time-dependent fields.

8 Conservation Laws

8.1 Core ideas

Fields carry energy, momentum, and angular momentum. Poynting's theorem describes local energy exchange between matter and fields. The Maxwell stress tensor expresses electromagnetic forces as momentum flux through surfaces.

For review, be able to derive Poynting's theorem, compute energy density and intensity, use stress tensor for pressure or force, and interpret field momentum. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

8.2 Mathematical spine

$$u = \frac{\epsilon_0 E^2}{2} + \frac{B^2}{2\mu_0}, \qquad \boldsymbol{S} = \frac{1}{\mu_0}\boldsymbol{E} \times \boldsymbol{B}, \qquad \partial_t u + \nabla \cdot \boldsymbol{S} = -\boldsymbol{J} \cdot \boldsymbol{E}$$

Section summary Conservation laws track energy and momentum flow in fields.

9 Electromagnetic Waves

9.1 Core ideas

In empty space Maxwell's equations imply self-propagating transverse waves moving at c. In media, waves refract, reflect, attenuate, and disperse depending on material response. Boundary conditions determine Fresnel coefficients and waveguide modes.

For review, be able to derive the wave equation, identify polarization and intensity, apply boundary conditions at interfaces, and relate index to phase velocity. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

9.2 Mathematical spine

$$\nabla^2 \boldsymbol{E} - \frac{1}{c^2}\partial_t^2 \boldsymbol{E} = 0, \qquad c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}, \qquad \boldsymbol{B} = \frac{1}{\omega}\boldsymbol{k} \times \boldsymbol{E}$$

Section summary Light is an electromagnetic wave predicted by Maxwell's equations.

10 Potentials and Fields

10.1 Core ideas

Scalar and vector potentials encode the fields with gauge freedom. Gauge transformations change potentials without changing $\boldsymbol{E}$ or $\boldsymbol{B}$. Lorenz gauge makes relativistic covariance and retarded solutions transparent; Coulomb gauge separates instantaneous constraint from radiation degrees of freedom.

For review, be able to use gauge transformations, derive retarded potentials, distinguish Lorenz and Coulomb gauges, and explain why potentials are not unique. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

10.2 Mathematical spine

$$\boldsymbol{B} = \nabla \times \boldsymbol{A}, \qquad \boldsymbol{E} = -\nabla V - \partial_t \boldsymbol{A}, \qquad V' = V - \partial_t \chi, \quad \boldsymbol{A}' = \boldsymbol{A} + \nabla \chi$$

Section summary Potentials reveal gauge freedom and causal propagation.

11 Radiation

11.1 Core ideas

Accelerated charges radiate. Far from the source, radiation fields fall as $1/r$, are transverse, and carry power through the Poynting flux. Dipole radiation is the leading approximation for many antennas and atoms, with angular patterns fixed by acceleration direction.

For review, be able to identify near and radiation zones, use Larmor power, estimate dipole radiation, and connect angular distribution to polarization. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

11.2 Mathematical spine

$$P = \frac{\mu_0 q^2 a^2}{6\pi c}, \qquad \langle P \rangle_{\text{dipole}} = \frac{\mu_0 \omega^4 p_0^2}{12\pi c}$$

Section summary Radiation is energy carried away by fields from accelerated charges.

12 Electrodynamics and Relativity

12.1 Core ideas

Electric and magnetic fields are frame-dependent parts of one electromagnetic field tensor. Lorentz transformations mix $\boldsymbol{E}$ and $\boldsymbol{B}$, while charge conservation becomes four-current conservation. Covariant notation makes Maxwell's equations compact and shows why magnetism is tied to relativity.

For review, be able to use four-vectors, transform fields between inertial frames, write Maxwell equations covariantly, and identify Lorentz invariants. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

12.2 Mathematical spine

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu, \qquad \partial_\mu F^{\mu\nu} = \mu_0 J^\nu, \qquad \partial_{[\alpha} F_{\beta\gamma]} = 0$$

Section summary Electromagnetism is naturally a relativistic field theory.