Complete Cosmology

Miko-pedia AI

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Contents

1 The Concordance Model of Cosmology

1.1 Core ideas

The concordance model, ΛCDM, describes a nearly flat universe with radiation, baryons, cold dark matter, dark energy, and nearly scale-invariant primordial perturbations. It is not a list of facts: it is a parameterized model connecting expansion, nucleosynthesis, CMB anisotropies, and galaxy clustering.

For review, be able to state the components of ΛCDM, define density parameters, explain flatness, and identify the key observational pillars. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

1.2 Mathematical spine

$$\Omega_i = \frac{\rho_i}{\rho_{\rm crit}}, \qquad \rho_{\rm crit} = \frac{3H_0^2}{8\pi G}, \qquad \sum_i \Omega_i \simeq 1$$

Section summary Modern cosmology is organized around the predictive ΛCDM model.

2 The Expanding Universe

2.1 Core ideas

Expansion is encoded in the scale factor $a(t)$. Cosmological redshift measures how wavelengths stretch with the universe. Distances are subtle because emission time, observation time, and geometry differ; comoving, angular-diameter, and luminosity distances answer different observational questions.

For review, be able to convert between scale factor and redshift, define Hubble rate, distinguish common cosmological distances, and interpret lookback time. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

2.2 Mathematical spine

$$1 + z = \frac{a_0}{a(t)}, \qquad H = \frac{\dot{a}}{a}, \qquad d_L = (1+z)^2 d_A$$

Section summary Expansion turns cosmic time into observable redshift and distance relations.

3 The Fundamental Equations of Cosmology

3.1 Core ideas

The Friedmann equations follow from applying Einstein's equation to a homogeneous and isotropic universe. Fluids dilute according to their equation of state: radiation as a^{-4}, matter as a^{-3}, and a cosmological constant as constant density.

For review, be able to derive density scaling, use Friedmann acceleration, compare radiation-, matter-, and dark-energy-dominated eras. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

3.2 Mathematical spine

$$H^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}, \qquad \dot{\rho} + 3H(\rho + p/c^2) = 0$$

Section summary Friedmann dynamics connects cosmic contents to expansion.

4 The Origin of Species

4.1 Core ideas

The early universe was hot and dense, so particle abundances were set by equilibrium, freeze-out, decays, and nuclear reactions. Big-bang nucleosynthesis predicts light elements; recombination releases the CMB; baryogenesis and dark matter freeze-out are deeper origin questions.

For review, be able to explain thermal equilibrium, freeze-out, nucleosynthesis, recombination, and why relic abundances probe early physics. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

4.2 Mathematical spine

$$\Gamma(T) \sim H(T) \quad \text{at freeze}-\text{out}, \qquad T \propto a^{-1}$$

Section summary Cosmic species are fossils of early-universe reaction rates.

5 The Inhomogeneous Universe: Matter and Radiation

5.1 Core ideas

Small perturbations grow into structure. Density contrast, velocity, gravitational potential, and radiation perturbations evolve differently before and after horizon entry. Photon-baryon acoustic oscillations leave signatures in the CMB and matter power spectrum.

For review, be able to define density contrast, explain horizon entry, describe acoustic oscillations, and read a matter power spectrum qualitatively. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

5.2 Mathematical spine

$$\delta(\boldsymbol{x}, t) = \frac{\rho(\boldsymbol{x}, t) - \bar{\rho}(t)}{\bar{\rho}(t)}, \qquad P(k) = \langle |\delta_{\boldsymbol{k}}|^2 \rangle$$

Section summary Structure formation starts from small coupled matter-radiation perturbations.

6 The Inhomogeneous Universe: Gravity

6.1 Core ideas

Gravity turns perturbations into growing structure. In Newtonian gauge, potentials source motion and lensing; in the subhorizon matter era, density perturbations obey a simple growth equation. Relativistic gauge issues matter on large scales.

For review, be able to use Poisson's equation for cosmological perturbations, solve qualitative growth, and distinguish physical perturbations from gauge artifacts. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

6.2 Mathematical spine

$$\ddot{\delta} + 2H\dot{\delta} - 4\pi G\bar{\rho}_m\delta = 0, \qquad \nabla^2\Phi = 4\pi G a^2\bar{\rho}_m\delta$$

Section summary Gravity amplifies primordial perturbations into cosmic structure.

7 Initial Conditions

7.1 Core ideas

Inflation explains why initial perturbations are nearly adiabatic, Gaussian, and scale invariant. The curvature perturbation is the central variable because it remains conserved on superhorizon scales for adiabatic modes. Its power spectrum seeds both CMB anisotropy and structure.

For review, be able to define adiabatic perturbations, read the scalar power spectrum, explain spectral tilt, and connect inflation to horizon and flatness problems. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

7.2 Mathematical spine

$$P_{\mathcal{R}}(k) = A_s\left(\frac{k}{k_*}\right)^{n_s-1}, \qquad n_s \approx 1$$

Section summary Initial conditions are encoded in the primordial curvature power spectrum.

8 Growth of Structure: Linear Theory

8.1 Core ideas

Linear theory describes perturbations while $|\delta| \ll 1$. Matter perturbations grow after matter-radiation equality, baryon acoustic features survive statistically, and transfer functions encode the processing of primordial fluctuations.

For review, be able to use growth factors, transfer functions, equality scale, and linear bias at a conceptual and formula level. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

8.2 Mathematical spine

$$P_m(k,z) = D^2(z)T^2(k)P_{\mathrm{prim}}(k), \qquad f = \frac{d\ln D}{d\ln a}$$

Section summary Linear theory links primordial fluctuations to large-scale structure.

9 The Cosmic Microwave Background

9.1 Core ideas

The CMB is radiation from last scattering. Its temperature anisotropies come from acoustic oscillations, gravitational redshift, Doppler motion, diffusion damping, and projection effects. Peak positions measure geometry; peak heights measure contents.

For review, be able to interpret the CMB angular power spectrum, name the Sachs-Wolfe and acoustic effects, and connect peaks to parameters. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

9.2 Mathematical spine

$$\frac{\Delta T}{T}(\hat{\boldsymbol{n}}) = \sum_{\ell m} a_{\ell m} Y_{\ell m}(\hat{\boldsymbol{n}}), \qquad C_\ell = \langle |a_{\ell m}|^2 \rangle$$

Section summary The CMB is a precision image of early-universe perturbations.

10 The Polarized CMB

10.1 Core ideas

CMB polarization is produced by Thomson scattering of radiation with a quadrupole anisotropy. Scalar perturbations generate E-modes; tensor perturbations can generate primordial B-modes. Lensing converts some E-mode power into B-modes.

For review, be able to distinguish E and B polarization, explain Thomson scattering origin, and state why B-modes are important. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

10.2 Mathematical spine

$$\text{scalar} \Rightarrow E, \qquad \text{tensor/lensing} \Rightarrow B, \qquad C_\ell^{EE}, C_\ell^{BB}$$

Section summary CMB polarization adds geometry and tensor information beyond temperature.

11 Probes of Structure I: Tracers

11.1 Core ideas

Galaxies, quasars, clusters, and the Lyman-alpha forest trace matter imperfectly. Bias, redshift-space distortions, selection functions, and shot noise must be modeled to infer the underlying density field.

For review, be able to define bias, correlation functions, redshift-space distortions, and the relation between tracers and matter. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

11.2 Mathematical spine

$$\delta_g = b\,\delta_m, \qquad \xi(r) = \langle \delta(\boldsymbol{x})\delta(\boldsymbol{x}+\boldsymbol{r})\rangle$$

Section summary Observed tracers are biased maps of the matter distribution.

12 Probes of Structure II: Gravitational Lensing

12.1 Core ideas

Lensing maps projected mass by measuring deflection, shear, convergence, and magnification. Weak lensing statistically measures large-scale structure; strong lensing probes dense systems; CMB lensing maps matter to high redshift.

For review, be able to define convergence and shear, connect lensing to projected potential, and distinguish weak and strong regimes. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

12.2 Mathematical spine

$$\kappa = \frac{\Sigma}{\Sigma_{\rm crit}}, \qquad \Sigma_{\rm crit} = \frac{c^2}{4\pi G}\frac{D_s}{D_l D_{ls}}$$

Section summary Lensing observes mass through its effect on light paths.

13 Probes of Structure III: Nonlinear Growth

13.1 Core ideas

On small scales, density contrasts become nonlinear. Spherical collapse, halo formation, virialization, N-body simulations, halo mass functions, and baryonic feedback describe the transition from smooth perturbations to galaxies and clusters.

For review, be able to explain nonlinear collapse, virial equilibrium, halo profiles, and why simulations are required. Keep the physical question visible: identify the degrees of freedom, the conserved quantities, the approximation being made, and the observable that would be measured.

13.2 Mathematical spine

$$2K + U = 0, \qquad \rho_{\rm NFW}(r) = \frac{\rho_s}{(r/r_s)(1+r/r_s)^2}$$

Section summary Nonlinear structure requires collapse physics and simulations.